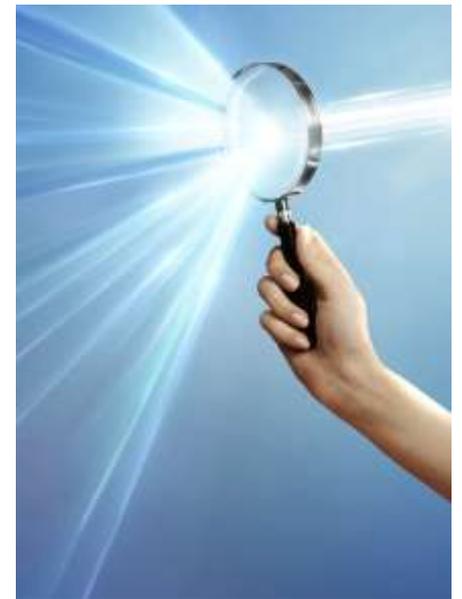


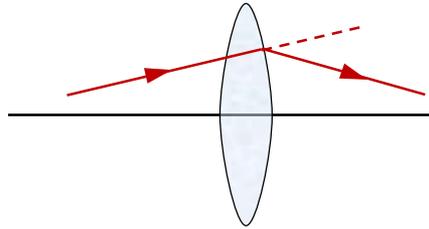
THIN LENSES



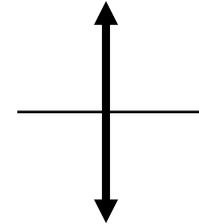
THIN LENSES

A lens is a transparent polished body bounded on two sides by curved surfaces. In particular, one of the surfaces may be plane. Lenses exist in a variety of places around us, from the interior of the human eye to the inner workings of computer memory systems. **Spherical lenses** have spherical surfaces as bounds.

Converging lenses convert a parallel beam of incident rays into a convergent beam. Converging lenses are biconvex, i.e. such that the thickness at the middle is larger than the thickness of edges (picture 1).

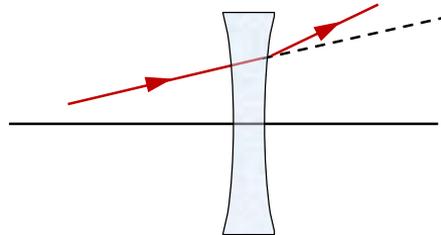


Picture 1

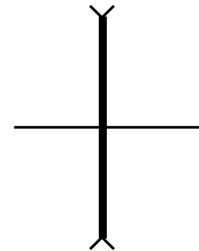


Picture 2: In a ray diagram, a biconvex lens is drawn as a vertical line with outward facing arrows to indicate the shape of the lens.

Diverging lenses convert a parallel beam of rays into a divergent beam. Diverging lenses are biconcave, i.e. such that the thickness at their edges is larger than the thickness at the middle (picture 3).

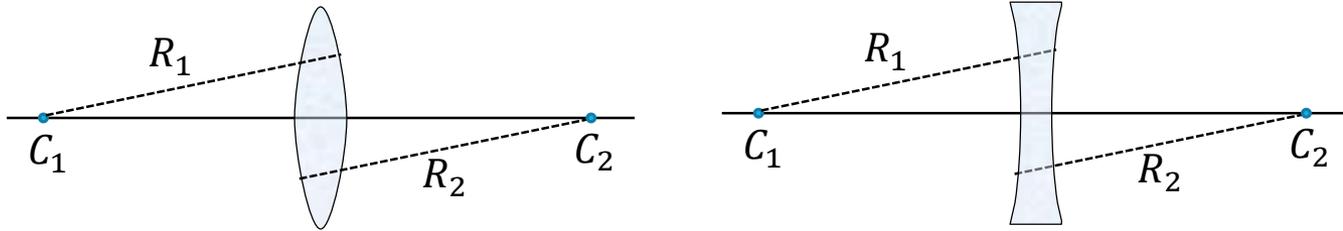


Picture 3



Picture 4: In a ray diagram, a biconcave lens is drawn as a vertical line with inward facing arrows to indicate the shape of the lens.

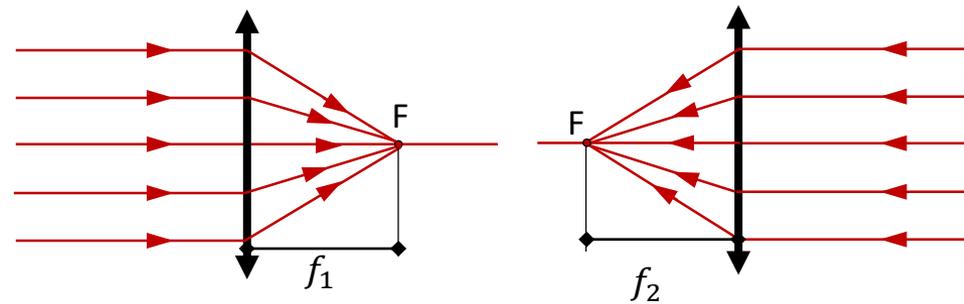
The straight line passing through the centres C_1 and C_2 of the spherical surfaces of a lens is called its **principal optical axis**. We shall consider only the special case of a **thin lens**—that is, a lens in which the thickest part is thin relative to the radii of curvature R_1 and R_2 of the two surfaces of the lens. (picture 5).



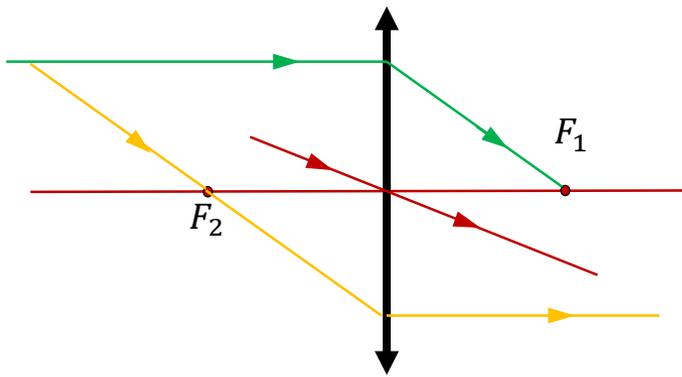
Picture 5

Image formation by a biconvex lens

A thin lens has *two* focal point (picture 6), one on each side of the lens. One focal point corresponds to parallel rays traveling from the left and the other corresponds to parallel rays traveling from the right. The distance from the centre of a lens to the focal point is called the **focal length (f)** of the lens.



Picture 6



Picture 7

To locate the image formed by a converging lens, the following three rays are drawn from the top of the object: (picture 7)

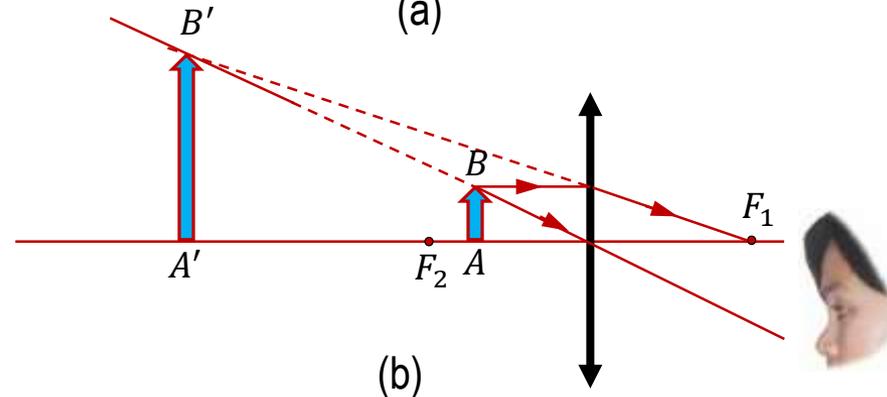
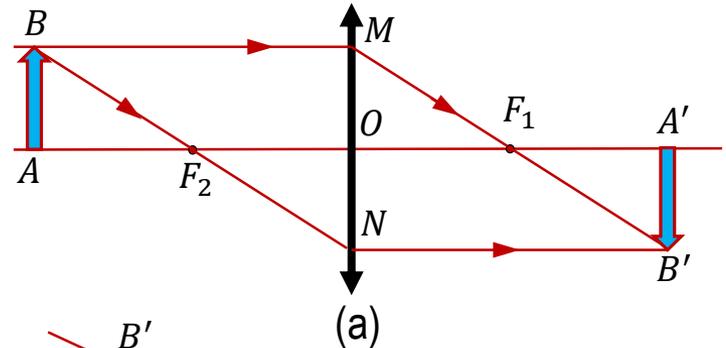
Ray 1 (green) The first ray is drawn parallel to the principal axis. After being refracted by the lens, this ray passes through (or appears to come from) one of the focal points.

Ray 2 (red) The second ray is drawn through the center of the lens. This ray continues in a straight line.

Ray 3 (yellow) The third ray is drawn through the other focal point and emerges from the lens parallel to the principal axis.

1. An object is between the focal point and the point at the double focal length. The image is real, reversed, and magnified, and lies on the other side of the lens (picture 8-a)

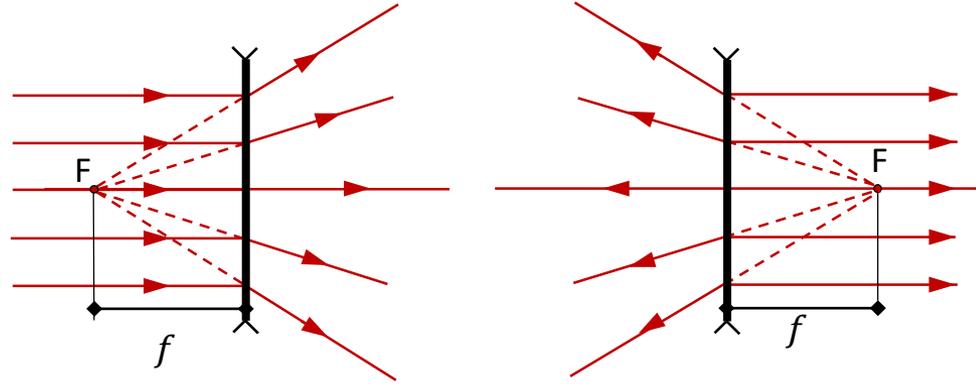
2. An object is between the focal point and the lens *center*. The image is virtual, erect, and magnified, and lies on the same side of the lens as the object is (picture 8-b)



Picture 8

Image formation by a biconcave lens

Picture 9 shows a thin biconcave lens. When rays that are parallel to the central axis of the lens are sent through this lens, these rays diverge, never passing through any common point, and so this lens is a diverging lens. However, extensions of the rays do pass through a common point F at a distance f from the center of the lens. Hence, the lens has a virtual focal point at F . Another virtual focus exists on the opposite side of the lens at F , symmetrically placed if the lens is thin.



Picture 9

Problems 1: Why does the focal length of a mirror not depend on the mirror material when the focal length of a lens does depend on the lens material?

Problems 2: In a Jules Verne novel, a piece of ice is shaped into a magnifying lens to focus sunlight to start a fire. Is that possible (picture 10)?

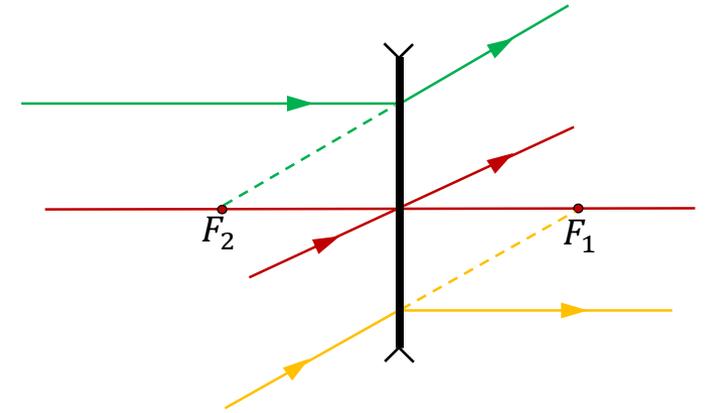


Picture 10

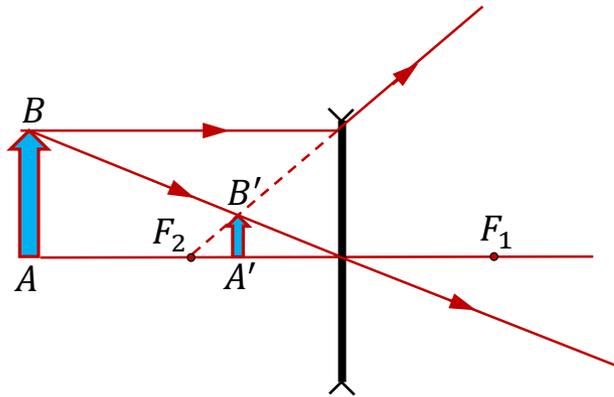
Ray 1 (green): Any incident ray traveling parallel to the principal axis of a diverging lens will refract through the lens and travel *in line with* the focal point (i.e., in a direction such that its extension will pass through the focal point).

Ray 2 (yellow): Any incident ray traveling towards the focal point on the way to the lens will refract through the lens and travel parallel to the principal axis.

Ray 3 (red): An incident ray that passes through the center of the lens will in effect continue in the same direction that it had when it entered the lens (picture 11).



Picture 11



Picture 12

The image formed by a *biconcave* lens is always virtual, erect, and diminished, and lies on the same side of the lens as the object does (picture 12).

THE LENS EQUATION

The lens equation expresses the quantitative relationship between the object distance (p), the image distance (l), and the focal length (f).

Using similarity of triangles ABF_2 i ONF_2 we can write:

$$\frac{AB}{ON} = \frac{AF_2}{OF_2} \Rightarrow \frac{AB}{ON} = \frac{p - f}{f}$$

Using similarity of triangles OMF_1 i $A'B'F_1$ we can write:

$$\frac{OM}{A'B'} = \frac{OF_1}{A'F_1} \Rightarrow \frac{OM}{A'B'} = \frac{f}{l - f}$$

If $AB = OM$ and $A'B' = ON$ then:

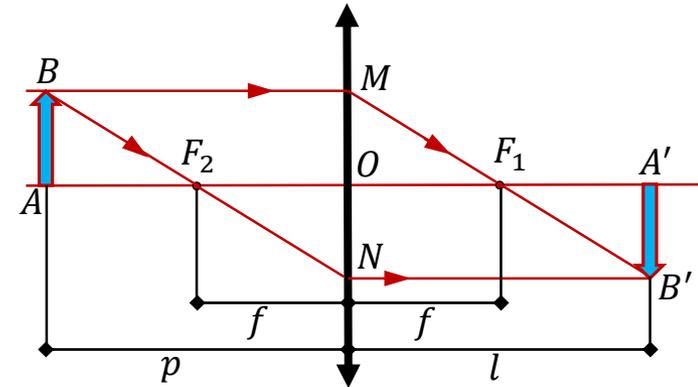
$$\frac{AB}{ON} = \frac{OM}{A'B'} \Rightarrow \frac{p - f}{f} = \frac{f}{l - f} \Rightarrow lf + pf = pl \Rightarrow \frac{1}{p} + \frac{1}{l} = \frac{1}{f}$$

The biconvex lens equation

◆ If the object is between the lens and focal point F then:

$$p < f \Rightarrow \frac{1}{p} > \frac{1}{f} \Rightarrow \frac{1}{l} = \frac{1}{f} - \frac{1}{p} < 0 \Rightarrow l < 0$$

A negative sign shows that the image is virtual.



Picture 13

For a biconcave lens, only a virtual image can be formed, regardless of the object's location on the central axis, so the image distance (l) and focal length (f) should be taken with sign $-$.

LATERAL MAGNIFICATION

The size of an object or image, as measured perpendicular to the lens's central axis, is called the object or image *height*. Let h represent the height of the object, and h' the height of the image. Then the ratio h'/h is called the **lateral magnification** m produced by the lens. However, by convention, the lateral magnification always includes a plus sign when the image orientation is that of the object and a minus sign when the image orientation is opposite that of the object.

For this reason, we write the formula for m as: $[m] = \frac{h'}{h}$

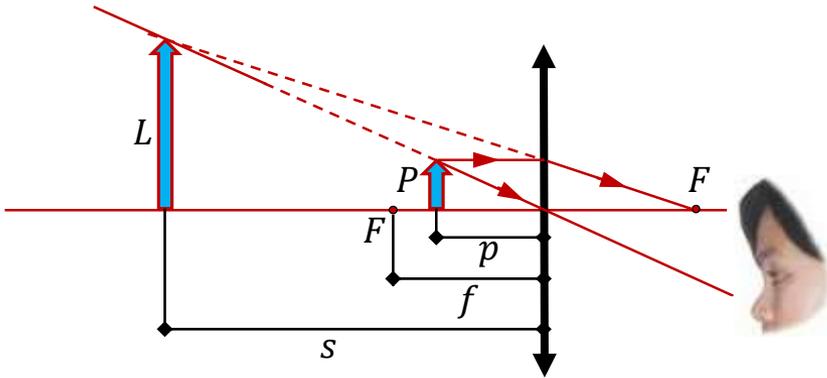
Magnification produced by the lens can also be write:  $m = \frac{l}{p}$

OPTICAL POWER

Optical power is the degree to which a **lens**, converges or diverges light. It is equal to the reciprocal of the focal length of the device:  $P = \frac{1}{f}$

High optical power corresponds to short focal length. The SI unit for optical power is the inverse metre (m^{-1}), which is commonly called the diopter (D). Converging lenses have positive optical power, while diverging lenses have negative power. When a lens is immersed in a refractive medium, its optical power and focal length change.

SIMPLE MAGNIFIER (LOUPE)



picture 14

The simple magnifier consists of a single biconvex lens. As the name suggests, this device increases the apparent size of an object.

Loupe is placed in front of biconvex lens, so the distance between virtual image and lens is $s = 25\text{cm}$.

Magnification of the loupe is: $\Rightarrow m = \frac{s}{p}$

Equation for biconvex lens is: $\frac{1}{p} - \frac{1}{s} = \frac{1}{f}$

$$\frac{1}{p} - \frac{1}{s} = \frac{1}{f} \Rightarrow u = \frac{s}{p} \Rightarrow u = s \cdot \left(\frac{1}{f} + \frac{1}{s} \right) \Rightarrow u = \frac{s}{f} + 1$$

A loupe is a small folding magnifier, typically 5X magnification.

PROBLEMS

1. A 5cm high object is placed 5 cm from a 15-cm focal length biconcave lens. Determine the image distance, the magnification of the image, the image height and the properties of the image?
2. The distance between an object and a biconvex lens is 5 times greater than the focal length of the lens. How many times will the image be smaller than the object?
3. A biconvex lens forms on a screen an image of a lamp magnified to twice its normal size. After the lens has been moved 36 cm closer to the screen it gives an image diminished by a factor of two. Find the optical power for biconvex lens?
4. The object is 16 cm away from the front focus of biconcave lens, and the real image 4 cm away from back focus. Find the distance between the object and his real image?
5. In a light-proof barrier there is a circular opening of radius 3 cm in which a biconvex lens of focal length 6cm is implanted. In front of the barrier, on the principal axis of the lens, there is a point light source P whose real image can be seen on the screen placed 16cm behind the barrier (see figure). What will be the radius of the bright circle on the screen if the lens is extracted from the barrier?

